EXACT SOLUTIONS OF EINSTEIN'S EQUATIONS FOR ENERGY DENSITIES OF SPECIAL TYPE

Yu.A.Tisenko[1]

[1] Chair of Physics, University "Our Ukraine", Dnipro, 49006, Ukraine

KEYWORDS

Einstein's equations; solutions; gravitation.

ABSTRACT

It was obtained exact solutions of Einstein's equations for energy densities of special type in this paper. It is considered cylindrically symmetrical interval. It is used cylindrical coordinates ρ, z, φ when calculating. Obtained solutions contain points of singularity, that form axis x^3 and plane $x^1 o x^2$. Surface of equal times is hyperboloid of two sheets, that does not cross coordinate axises and plane $x^1 o x^2$.

I. INTRODUCTION

It was obtained exact solutions of Einstein's equations for energy densities of special type in this paper. It is considered cylindrically symmetrical interval. It is used cylindrical coordinates ρ, z, φ when calculating. Obtained solutions contain points of singularity, that form axis x^3 and plane $x^1 o x^2$. Surface of equal times is hyperboloid of two sheets, that does not cross coordinate axises and plane $x^1 o x^2$.

Results of this paper have physical interest by the next reasons:

1. It was shown in this paper, that at limit $\Lambda \to 0$ gravitational field may exist even in absolute vacuum ($w\ (\rho, z) = 0$).
2. In this paper it was shown, that gravitational field in absolute vacuum may have surface of equal times and equal lengths of hyperboloid type.

II. RICHI'S TENSOR

Einstein's equations are [1]

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \Lambda g_{\mu\nu} = \varkappa T_{\mu\nu}, \tag{1}$$

$$\varkappa = \frac{8\pi G}{c^4}.$$

$T_{\mu\nu}$ is momentum-energy tensor: $T_{oo} = w\ (\rho,z)$, other components are equal to zero. For physical calculations they use solutions in the limit $\Lambda \to 0$ [22, 23]. In case of axial symmetry we look for solutions of equations (1) of such a type:

$$ds^2 = A_0(\rho,z)dx^{o2} + A_1(\rho,z)d\rho^2 + A_2(\rho,z)\rho^2 d\varphi^2 + A_3(\rho,z)dz^2.$$

So, we look only for solutions, which correspond to diagonal form of ds^2. Functions A_i do not depend on cylindrical coordinate φ. Distortion of length for direction $\rho d\varphi$ is absent: $A_2(\rho,z) = -1$.

Metric tensor components are equal to

$$g_{00} = A_0(\rho, z);$$

$$g_{11} = A_2(\rho, z) + B(\rho, z)\cos^2\varphi;$$

$$g_{12} = g_{21} = B(\rho, z)\sin\varphi \cdot \cos\varphi;$$

$$g_{22} = A_2(\rho, z) + B(\rho, z)\sin^2\varphi;$$

$$g_{33} = A_3(\rho, z);$$

$$B(\rho, z) = A_1(\rho, z) - A_2(\rho, z).$$

Metric tensor with upper indexes components are equal to

$$g^{00} = A_0^{-1}(\rho, z);$$

$$g^{11} = \frac{A_2(\rho, z) + B(\rho, z)\sin^2\varphi}{A_1(\rho, z) \cdot A_2(\rho, z)};$$

$$g^{12} = g^{21} = -\frac{B(\rho, z)\sin\varphi\cos\varphi}{A_1(\rho, z) \cdot A_2(\rho, z)};$$

$$g^{22} = \frac{A_2(\rho, z) + B(\rho, z)\cos^2\varphi}{A_1(\rho, z) \cdot A_2(\rho, z)};$$

$$g^{33} = A_3^{-1}(\rho, z).$$

All other components of tensors $g_{\mu\nu}$ and $g^{\mu\nu}$ are equal to zero.

After calculation of Kristoffel's symbols $\Gamma_{\nu\rho\sigma}$ and $\Gamma^{\nu}{}_{\rho\sigma}$ and definition of tensor of crookedness $R_{\mu\nu\rho\sigma}$, [2-5], we determine components of Richi's tensor $R_{\mu\nu}$:

$$R_{00} = -\frac{1}{2A_1}\left\{\left(\partial_\rho^2 A_0\right) - \frac{1}{2}(\partial_\rho A_0)\left[\left(\partial_\rho ln\left|\frac{A_0 A_1}{A_3}\right|\right) - \frac{2}{\rho}\right]\right\} -$$

$$- \frac{1}{2A_3} \cdot \left\{(\partial_z^2 A_0) - \frac{1}{2}(\partial_z A_0)\left(\partial_z ln\left|\frac{A_0 A_3}{A_1}\right|\right)\right\};$$

$$R_{11} = D_1 cos^2\varphi + D_2 sin^2\varphi;$$

$$R_{12} = R_{21} = (D_1 - D_2) sin\,\varphi\, cos\,\varphi;$$

$$R_{13} = R_{31} = D_3 \cdot cos\,\varphi\,;$$

$$R_{23} = R_{32} = D_3 \cdot sin\,\varphi\,;$$

$$R_{22} = D_1 sin^2\varphi + D_2 cos^2\varphi;$$

$$R_{33} = -\frac{1}{2A_0}\,(\partial_z^2 A_0) - \frac{1}{2A_1}\,(\partial_z^2 A_1) - \frac{1}{2A_1}\,\left(\partial_\rho^2 A_3\right) + \frac{1}{4}\,(\partial_z ln|A_0|) \times$$

$$\times (\partial_z ln|A_0 A_3|) + \tfrac{1}{4}(\partial_z ln|A_1|)\ \cdot (\partial_z\, ln|A_1 A_3|) + \tfrac{1}{4A_1}(\partial_\rho\, A_3) \times$$

$$\times \left[\left(\partial_\rho ln\, \left|\frac{A_1\, A_3}{A_0}\right|\right) - \frac{1}{\rho}\right].$$

Other components of tensor $R_{\mu\nu}$ are equal to zero. Functions $D_i\ (\rho, z)$are equal to

$$D_1\ (\rho, z) = -\frac{1}{2A_0}\left(\partial_\rho^2 A_0\right) - \frac{1}{2A_3}\,(\partial_z^2 A_1) - \frac{1}{2A_3}\,\left(\partial_\rho^2 A_3\right) + \frac{1}{4}\,(\partial_\rho\, ln|A_0|)\ \times$$

$$\times \left(\partial_\rho\, ln|A_0 A_1|\right) + \frac{1}{4}\,\left(\partial_\rho\, ln|A_3|\right) \cdot \left(\partial_\rho\, ln|A_1 A_3|\right) + \frac{1}{4A_3}\,(\partial_z A_1)(\partial_z\, ln\left|\frac{A_1\, A_3}{A_0}\right|) +$$

$$+ \frac{1}{2}\,\left(\partial_\rho\, ln|A_1|\right) \cdot \frac{1}{\rho}\ ;$$

$$D_2\ (\rho, z) = \frac{1}{2A_1}\left(\partial_\rho\, ln\left|\frac{A_0\, A_3}{A_1}\right|\right) \cdot \frac{1}{\rho}\ ;$$

$$D_3\ (\rho, z) = -\frac{1}{2A_0}\left(\partial_{\rho z}^2 A_0\right) + \frac{1}{4}\,\left(\partial_\rho\, ln|A_0|\right) \cdot (\partial_z\, ln|A_0 A_1|) + \frac{1}{4}\,(\partial_z\, ln|A_0|) \times$$

$$\times \left(\partial_\rho\, ln|A_3|\right) + \frac{1}{2}\,(\partial_z\, ln|A_1|) \cdot \frac{1}{\rho}.$$

We determine scalar crookedness from equations (1) (Λ = 0):

$$R = -A_0^{-1} \varkappa w.$$

Now equations (1) take the form

$$R_{00} - \frac{1}{2}\,\varkappa w \;= 0; \qquad (2)$$

$$D_1 + \frac{1}{2}\,\varkappa w \cdot \frac{A_1}{A_0} = 0; \qquad (3)$$

$$D_2 - \frac{1}{2}\,\varkappa w \cdot \frac{1}{A_0} = 0; \qquad (4)$$

$$D_3 = 0; \qquad (5)$$

$$R_{33} + \frac{1}{2}\,\varkappa w \cdot \frac{A_3}{A_0} = 0. \qquad (6)$$

Other equations from system (1) turn into identities.

We shall use new functions:

$$Q_1\,(\rho, z) = -\frac{1}{2A_0}\left(\partial_\rho^2 A_0\right) + \frac{1}{4A_0}\,(\partial_\rho\, A_0) \cdot \left[\left(\partial_\rho\, ln\left|\frac{A_0\,A_1}{A_3}\right|\right) - \frac{2}{\rho}\right];$$

$$Q_2\,(\rho, z) = -\frac{1}{2A_0}(\partial_z^2 A_0) + \frac{1}{4A_0}(\partial_z\, A_0) \cdot \left(\partial_z\, ln\left|\frac{A_0\,A_3}{A_1}\right|\right);$$

$$Q_3\,(\rho, z) = -\frac{1}{2A_0}\,\left(\partial_\rho^2 A_0\right) - \frac{1}{2A_3}\,\left(\partial_\rho^2 A_3\right) + \frac{1}{4}\,\left(\partial_\rho\, ln|A_0|\right)\,\cdot\left(\partial_\rho\, ln|A_0 A_1|\right) +$$

$$+\frac{1}{4}\,\left(\partial_\rho\, ln|A_3|\right)\cdot\,\left(\partial_\rho\, ln|A_1 A_3|\right) + \frac{1}{2}\,\left(\partial_\rho\, ln|A_1|\right) \cdot\, \frac{1}{\rho};$$

$$Q_4\,(\rho, z) = -\frac{1}{2A_1}(\partial_z^2 A_1) + \frac{1}{4}\,(\partial_z\, ln|A_1|) \cdot\,\left(\partial_z\, ln\left|\frac{A_1\,A_3}{A_0}\right|\right);$$

$$Q_5\,(\rho, z) = \frac{1}{2}\left(\partial_\rho\, ln\left|\frac{A_0\,A_3}{A_1}\right|\right) \cdot \frac{1}{\rho}\;;$$

$$Q_6\,(\rho, z) = -\frac{1}{2A_0}\,(\partial_z^2 A_0) - \frac{1}{2A_1}\,(\partial_z^2 A_1) + \frac{1}{4}\,(\partial_z\, ln|A_0|)\;\times$$

$$\times \ (\partial_z \, ln|A_0 A_3|) + \frac{1}{4} \ (\partial_z \, ln|A_1|) \ \cdot \ (\partial_z \, ln|A_1 A_3|) \ ;$$

$$Q_7 \ (\rho, z) = -\frac{1}{2A_3} \ \left(\partial_\rho^2 A_3\right) \ + \frac{1}{4} \ \left(\partial_\rho \, ln|A_3|\right) \cdot \left[\left(\partial_\rho \, ln \left|\frac{A_1 \, A_3}{A_0}\right|\right) - \frac{1}{\rho}\right].$$

Then we have

$$R_{00} = \frac{A_0}{A_1} Q_1 + \frac{A_0}{A_3} \ Q_2 \ ;$$

$$D_1 = Q_3 + \frac{A_1}{A_3} \ Q_4 \ ;$$

$$D_2 = \frac{1}{A_1} \ Q_5 \ ;$$

$$R_{33} = Q_6 + \frac{A_3}{A_1} \ Q_7 \, .$$

Now after a number of transformations we write down equations (2) – (6) in the form

$$Q_2 \ (Q_3 + Q_5) + Q_4 (Q_5 - Q_1) = 0; \qquad (7)$$

$$\frac{A_1}{A_3} \ Q_2 \ = (Q_5 - Q_1); \qquad (8)$$

$$2 \ \frac{A_0}{A_1} \ Q_5 = \varkappa w; \qquad (9)$$

$$D_3 = 0; \qquad (10)$$

$$Q_6 \ (Q_5 - Q_1) + Q_2 (Q_7 + Q_5) = 0. \qquad (11)$$

Then we cross to new functions $f_o \ (\rho,z)$, $f_1(\rho,z)$ and $f_3(\rho,z)$:

$$A_0 = e^{f_o}; \qquad A_1 = -e^{f_1}; \qquad A_3 = -e^{f_3}.$$

Then all functions Q_i and D_3 in equations (7) – (11) will depend only on functions f_o, f_1, f_3 derivatives and on coordinate ρ:

$$Q_1 = -\frac{1}{2}\, f_{o\rho\rho} - \frac{1}{4} f_{o\rho} \left[\left(f_{o\rho} - f_{1\rho} + f_{3\rho} \right) + \frac{2}{\rho} \right];$$

$$Q_2 = -\frac{1}{2} f_{ozz} - \frac{1}{4} f_{oz} \left(f_{oz} + f_{1z} - f_{3z} \right);$$

$$Q_3 = -\frac{1}{2}\, f_{o\rho\rho} - \frac{1}{2}\, f_{3\rho\rho} - \frac{1}{4} f_{o\rho}^2 - \frac{1}{4} f_{3\rho}^2 + \frac{1}{4} f_{1\rho} \left(f_{o\rho} + f_{3\rho} + \frac{2}{\rho} \right);$$

$$Q_4 = -\frac{1}{2} f_{1zz} + \frac{1}{4} f_{1z} \left(f_{3z} - f_{oz} - f_{1z} \right);$$

$$Q_5 = \frac{1}{2} \left(f_{o\rho} + f_{3\rho} - f_{1\rho} \right) \cdot \frac{1}{\rho};$$

$$Q_6 = -\frac{1}{2} f_{ozz} - \frac{1}{2} f_{1zz} - \frac{1}{4} f_{oz}^2 - \frac{1}{4} f_{1z}^2 + \frac{1}{4} \left(f_{oz} + f_{1z} \right) \cdot f_{3z};$$

$$Q_7 = -\frac{1}{2} f_{3\rho\rho} - \frac{1}{4} f_{3\rho} \left(f_{o\rho} - f_{1\rho} + f_{3\rho} + \frac{1}{\rho} \right);$$

$$D_3 = -\frac{1}{2} f_{o\rho z} - \frac{1}{4} f_{oz} \left(f_{o\rho} - f_{3\rho} \right) + \frac{1}{4} f_{1z} \left(f_{o\rho} + \frac{2}{\rho} \right).$$

We use marks: $f_{o\rho} = \partial_\rho f_o$, $f_{1z} = \partial_z f_1$, $f_{o\rho z} = \partial_{\rho z}^2 f_o$ and so on.

Now we have to solve equations (7) – (11) relatively functions f_k. We look for solutions of such a form:

$$f_o = f_o(X); \qquad f_1 = f_1(X); \qquad f_3 = f_3(X);$$

$$X = \alpha \rho z; \qquad X \geq 0.$$

α is dimensional constant: $[\alpha] = \frac{1}{m^2}$.

III. SOLUTIONS OF EINSTEIN'S EQUATIONS

We shall use new functions:

$$Q_1 = Q_1^{(0)} \cdot \frac{1}{\rho^2}; \qquad Q_2 = Q_2^{(0)} \cdot \rho^2; \qquad Q_3 = Q_3^{(0)} \cdot \frac{1}{\rho^2};$$

$$Q_4 = Q_4^{(0)} \cdot \rho^2; \qquad Q_5 = Q_5^{(0)} \cdot \frac{1}{\rho^2}; \qquad Q_6 = Q_6^{(0)} \cdot \rho^2; \qquad (12)$$

$$Q_7 = Q_7^{(0)} \cdot \frac{1}{\rho^2}; \qquad D_3 = D_3^{(0)}.$$

All functions $Q_i^{(0)}$, $D_3^{(0)}$ depend only on one new variable $X = \alpha\rho z$. Now we rewrite equations (7) – (11) in such a form:

$$Q_2^{(0)}\left(Q_3^{(0)} + Q_5^{(0)}\right) + Q_4^{(0)}\left(Q_5^{(0)} - Q_1^{(0)}\right) = 0; \qquad (13)$$

$$\frac{A_1}{A_3}\, Q_2^{(0)} \cdot \rho^2 = \left(Q_5^{(0)} - Q_1^{(0)}\right) \cdot \frac{1}{\rho^2}; \qquad (14)$$

$$2\, \frac{A_0}{A_1} \cdot Q_5^{(0)} \cdot \frac{1}{\rho^2} = \varkappa w; \qquad (15)$$

$$D_3^{(0)} = 0; \qquad (16)$$

$$Q_6^{(0)}\left(Q_5^{(0)} - Q_1^{(0)}\right) + Q_2^{(0)}\left(Q_7^{(0)} + Q_5^{(0)}\right) = 0. \qquad (17)$$

Left parts of equations (13), (16), (17) contain quantities, that depend only on X. Equation (15) determines energy density $w(\rho, z)$.

Equation (14) contains “unnecessary” multipliers ρ^2 and $\frac{1}{\rho^2}$. That is why equation (14) may be satisfied only by conditions

$$Q_2^{(0)} = 0; \qquad Q_5^{(0)} - Q_1^{(0)} = 0.$$

By that conditions equations (13) and (17) will be also satisfied automatically.

So, we rewrite equations (13) – (17) in such a form:

$$Q_2^{(0)} = 0; \tag{18}$$

$$Q_5^{(0)} - Q_1^{(0)} = 0; \tag{19}$$

$$2\,\frac{A_0}{A_1}\,Q_5^{(0)} \cdot \frac{1}{\rho^2} = \varkappa \cdot w; \tag{20}$$

$$D_3^{(0)} = 0. \tag{21}$$

Using formulas (12), we calculate functions $Q_i^{(0)}$ and $D_3^{(0)}$, that are contained in equations (18) – (21):

$$Q_2^{(0)} = \left[-\frac{1}{2}f_o'' - \frac{1}{4}f_o'(f_o' + f_1' - f_3')\right] \cdot \alpha^2;$$

$$Q_1^{(0)} = \left[-\frac{1}{2}f_o'' - \frac{1}{4}f_o'(f_o' - f_1' + f_3')\right] \cdot X^2 - \frac{1}{2}f_o' \cdot X;$$

$$Q_5^{(0)} = \frac{1}{2}(f_o' + f_3' - f_1') \cdot X;$$

$$D_3^{(0)} = \left[-\frac{1}{2}f_o'' - \frac{1}{4}f_o'(f_o' - f_3') + \frac{1}{4}f_o' \cdot f_1'\right] \cdot \alpha X - \frac{1}{2}(f_o' - f_1') \cdot \alpha\,.$$

Touch means variable *X* differentiation.

From equation (18) we find:

$$f_1' - f_3' = -2\frac{f_o''}{f_o'} - f_o' \tag{22}$$

and substitute this result to equation (19). We have:

$$f_o''(1 + f_o'X) + \frac{3}{2}\,(f_o')^2 + \frac{1}{2}(f_o')^3 \cdot X = 0. \tag{23}$$

Two solutions of equation (23) have the form

$$f_o = a_0\, ln\frac{X}{|c_0|}; \qquad a_{01} = 1; \qquad a_{02} = -2. \tag{24}$$

We shall transform equation (23) to the system of first order equations:

$$f_o'(X) = y(X); \tag{23a}$$

$$y'(X) \cdot (1 + y(X) \cdot X) + \tfrac{3}{2}\,\big(y(X)\big)^2 + \tfrac{1}{2}\big(y(X)\big)^3 \cdot X = 0\,. \tag{23b}$$

Solution

$$\varphi_i(X) = \begin{pmatrix} a_0 ln\dfrac{X}{|c_0|} \\ \dfrac{a_0}{X} \end{pmatrix}_i$$

of system (23a) – (23b) is unstable solution. In order to prove that we shall consider series of system (23a) – (23b) solutions

$$v_i(X) = \begin{pmatrix} c' \\ 0 \end{pmatrix}_i .$$

We see, that

$$|v_1(X) - \varphi_1(X)| = \left|c' - a_0\, ln\frac{X}{|c_0|}\right| \nless \varepsilon$$

for all $X \in [X_0,\infty)$. So, solution $\varphi_i(X)$ is unstable one.

Solution $\varphi_i(X)$ unstability is explained by such a fact: system (23a) – (23b) has different serieses of solutions, which correspond to gravitational fields of different configurations.

Now from equations (21) and (22) we find:

$$f_1 = a_1\, ln\frac{X}{|c_1|}; \qquad f_3 = a_3\, ln\frac{X}{|c_3|}; \qquad (25)$$

$$a_1 = \frac{a_0}{(a_0+1)} = \begin{cases} \frac{1}{2}, \\ 2\,; \end{cases} \qquad a_3 = \frac{a_0^2-2}{(a_0+1)} = \begin{cases} -\frac{1}{2}, \\ -\,2\,. \end{cases}$$

So, solutions of Einstein's equations (1) have the form

$$A_0 = \left(\frac{X}{|c_0|}\right)^{a_0}; \quad A_1 = -\left(\frac{X}{|c_0|}\right)^{a_1}; \quad A_2 = -1; \quad A_3 = -\left(\frac{X}{|c_0|}\right)^{a_3}; \quad (26)$$

$$X = \alpha\rho z; \qquad X \geq 0.$$

If "traveller" is at the same hyperboloid $X = |c_0|$, where "observer" is, then there will not be distortion of lengths for "observer". That is why we put $|c_0| = |c_1| = |c_3|$ in (26). Both solutions ($a_{01} = 1$ and $a_{02} = -2$) have singularities [20-21] when $X = 0$, that is at axis x^3 and at plane x^1ox^2.

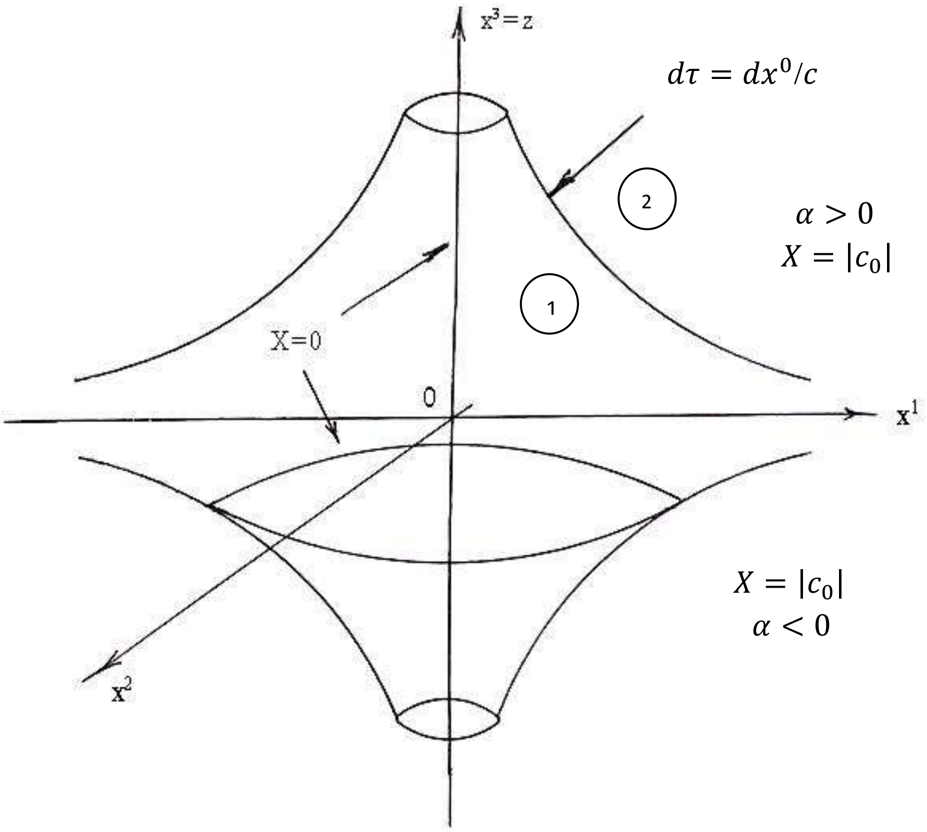
Energy density is determined by equation (20):

$$\varkappa w = \begin{cases} 0 & if\ a_0 = 1; \\ 6\,\frac{|c_0|^4}{X^4}\cdot\frac{1}{\rho^2} & if\ a_0 = -2. \end{cases}$$

<u>First solution.</u> Phenomenon of gravitational field existence in absolute vacuum was formerly examined in works [17-19]. Vacuum solutions of Einstein's equations are Miln's model and De Sitter's models [17]. In work [18] it was indicated that such a gravitational field, perhaps, is formed by superheavy virtual particles, that arise in vacuum in boundaries of quantum

indefinability [6-16]. In this paper it was shown, that gravitational field in absolute vacuum may have surface of equal times and equal lengths of hyperboloid type.

For first solution time interval is $d\tau = \sqrt{\frac{X}{|c_0|}} \cdot dx^0/\, c$; “traveller” time is accelerated out of hyperboloid $X = |c_0|$, if “traveller” moves in direction of X increase and time slows down, if it moves in direction of X decrease. When X=0, then “traveller” time stops. At hyperboloid $X = |c_0|$ $d\tau = dx^0/c$.



1 – domain of “traveller” time slowing-down; 2 – domain of “traveller” time acceleration.

Picture 1. Hyperboloid of equal times ($d\tau = dx^0/c$) for first solution.

<u>Second solution.</u> Time interval is $d\tau = \frac{|c_0|}{X}\, dx^0/c$; "traveller" time slows down if "traveller" moves out of hyperboloid $X = |c_0|$ in direction of X increase and time is accelerated if it moves in direction of X decrease. At hyperboloid $d\tau = dx^0/c$. $X = 0$ are points of singularity – axis x^3 and plane x^1ox^2. When $X \to 0$, then $\frac{d\tau}{dx^0}c \to \infty$. For second solution domains 1 and 2 at picture 1 exchange their places.

For second solution we obtain formula for "observer" ($X = |c_0|$) distance to singularity axis x^3:

$$\rho_o = \left(\frac{6}{\varkappa w}\right)^{1/2}.$$

If quantity of energy density w is too small, then singularity axis is at very large distance from "observer".

We shall write down also solution of equations (1) for $\Lambda \neq 0$. We take into consideration that term Λ modifies (0,0), (1,1), (1,2), (2,1), (2,2), (3,3) components of the momentum-energy tensor in equations (1). We shall consider new expression, that contains energy density:

$$W\,(\rho, z) = w(\rho, z) - \frac{2\Lambda \cdot A_0(\rho, z)}{\varkappa}.$$

Component T_{oo} of momentum-energy tensor may be written in the form

$$T_{oo} = W + \frac{2\Lambda A_0}{\varkappa}. \qquad (27)$$

We shall consider $\frac{2\Lambda A_0}{\varkappa}$ in formula (27) as a small indignation of component T_{oo}:

$$W \gg \frac{2\Lambda A_0}{\varkappa}. \qquad (28)$$

Then for zeroth-order approach ($T_{oo} = W$) solution of equations (1), when $\Lambda \neq 0$, coincides completely with exact solution of equations (1) for $\Lambda = 0$, when $a_0 = -2$. That is, zeroth-order solution, when $\Lambda \neq 0$, is determined by formulas (26) for $a_0 = -2$ under condition (28). And energy density w for zeroth-order solution is given by formula

$$\varkappa w = 2\,\frac{|C_0|^2}{X^2}\left(3 \cdot \frac{|C_0|^2}{X^2} \cdot \frac{1}{\rho^2} + \Lambda\right), \quad \text{if} \quad 3 \cdot \frac{|C_0|^2}{X^2} \cdot \frac{1}{\rho^2} \gg \Lambda.$$

Taking into consideration small quantity of Λ, we infer that zeroth-order solution (26) ($a_0 = -2$) is true for large interval of variables X and ρ significances.

IV. SUMMARY

So, in this paper it was shown, that at limit $\Lambda \to 0$ gravitational field may exist even in absolute vacuum ($w\ (\rho, z) = 0$). It was shown in this paper, that gravitational field in absolute vacuum may have surface of equal times and equal lengths of hyperboloid type. We have also determined points of singularity disposition for energy densities of special type in this paper.

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